

# Maximum Likelihood Approach for Tracking Multiple Mobile Agents with A Moving Doppler Radar System

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**Abstract**—Tracking mobile agents with a Doppler radar system mounted on a moving vehicle is considered in this paper. Dopplers modulated from mobile agents on the single frequency continuous wave signals are analyzed in order to estimate the positions and velocities of multiple mobile agents. The measurement noise is assumed to be Gaussian and the maximum likelihood estimation is utilized to enhance the localization accuracy.

## I. INTRODUCTION

Tracking multiple moving agents can have many potential applications in situations such as, security surveillance, disaster search, rescue missions and urban warfare [1]–[13]. In the automobile industry, this technology can be used in a driver alerting systems to provide the driver with helpful information about the surrounding of the vehicle. Improved safety functions are now introduced to vehicles to assist the driver and to reduce the risk of accidents. Key points of improved operations are effectiveness and the information content in the perception of the surroundings.

Radar technology is emerging as a strong candidate in the context of sensing the surroundings in automobile applications [14]–[16]. The use of ultra-wideband radar for short range vehicular applications is discussed in [15]. In [16], fusing the vision data with radar information to enhance the detection accuracy has been studied. The approach in [14] introduces a novel Doppler sensor architecture for vehicular applications.

Continuous Wave (CW) radar systems compared to other types of radar techniques available today are the simplest to implement [17]. Single frequency CW radar can measure Doppler frequency shifts with a high degree of accuracy. However, this method is not generally used in range measurements. In order to measure the range, more sophisticated systems derived from CW radar are currently being used. These systems are costly, not user friendly and require complex hardware to implement. CW radar has superior clutter suppression capabilities and therefore, it can be used to detect moving agents in strong clutter [1]. Frequency Modulated CW (FMCW) radar and pulsed Doppler radar both evolved from the CW radar technique. They are capable of detecting range albeit prone to clutter requiring clutter mitigation algorithms.

CW single frequency radar consisting of two antenna elements to find the direction of arrival (DOA) has been explored

in [1]. In this system the DOA of a mover is measured using the phase difference of reflected waves arriving at the receiving antennas. Doppler separations of the moving agents should be adequately high for this system to track multiple agents accurately. Furthermore, range or velocity measurements cannot be obtained for agents using this system.

In [2], localization is carried out by two-frequency Doppler and DOA radar systems. However, this method is not capable of finding the velocities of agents. Also in [3], [18] and [17], only the location information of the agents could be obtained. Another approach is explored in [19] and [20], but the performance degrades rapidly as the number of moving agents increases.

For the above systems discussed, only the location information of agents can be derived and the time derivative of the displacement is used to find the velocities of the agents. If the agents change their velocities rapidly, these systems fail to map the exact dynamic behavior of the mobile agents.

In [21], two receiving elements are kept half a wave length apart and the other two receiving elements are also placed with the same distance apart. These two sets are placed apart from each other, in line, facing the mobile agents. The bearing of an agent with respect to a particular set of elements can be found by measuring the phase difference of the reflected waves. Similarly, the agent's bearing with respect to the other two set of receiving elements can be found in the same manner. Using these parameters, the agent's location can be tracked. The agent's velocity can be derived by the Doppler shifts formed by the agent in the direction of sensors. In many cases, multiple mobile agents give rise to different Doppler shifts. By processing data via Doppler discrimination, positions of different agents can be found by measuring phase differences at each Doppler frequency bin. This method can be utilized in vehicular applications to sense its surroundings with the knowledge of the vehicle dynamics.

Goal of this paper is to provide a method to track multiple mobile agents from an array of Doppler sensors mounted on a moving vehicle. This paper has been organized in the following way. First section describes the basic theory relevant to Doppler radar and the second section introduces the theory in vehicle dynamics. Utilizing the maximum likelihood

estimation for localization is discussed in the third section and finally the simulation results are provided.

## II. BASIC THEORY FOR DOPPLER BASED LOCALIZATION

Consider two nearby scattered waves returning from a mobile agent as shown in Fig.1. These two waves can be considered as parallel when the distance to the mobile agent is considerably higher than the distance between the sensors. Two antenna elements,  $Rx_1$  and  $Rx_2$  receive these two signals with a path difference of  $y'$ , due to the relative position of the agent.

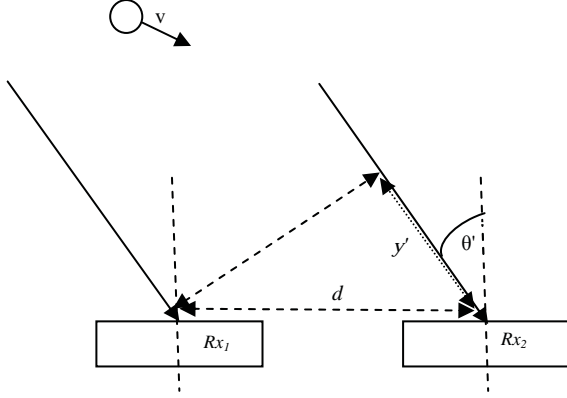


Fig. 1. DOA using two receiving elements

$$y' = d \sin \theta', \quad (1)$$

where,  $d$  is the distance between the two antenna elements  $Rx_1$  and  $Rx_2$ , and  $\theta'$  is the Direction of Arrival (DOA) of the mobile agent. The phase difference of the two waves  $\Delta\psi$ , can be written as,

$$\Delta\psi = \frac{2\pi y'}{\lambda_c}, \quad (2)$$

where,  $\lambda_c$  is the wave length of the carrier frequency. Then from (1) and (2), the DOA can be expressed as,

$$\theta' = \arcsin\left(\frac{\Delta\psi \lambda_c}{2\pi d}\right). \quad (3)$$

Consider a mobile agent  $T$ , at coordinates  $(x,y)$  with velocity towards  $x$ -direction and  $y$ -direction to be  $u_x$  and  $u_y$  respectively (Fig.2). Here,

$A(p, 0)$  = Two antenna elements at coordinates  $(p, 0)$ ,

$B(q, 0)$  = Two antenna elements at coordinates  $(q, 0)$ ,

$v_A$  = Radial velocity of agent, towards  $(p, 0)$  measured by Doppler,

$v_B$  = Radial velocity of agent, towards  $(q, 0)$  measured by Doppler,

$\theta_A$  = Bearing of agent with respect to  $A$ ,

$\theta_B$  = Bearing of agent with respect to  $B$ .

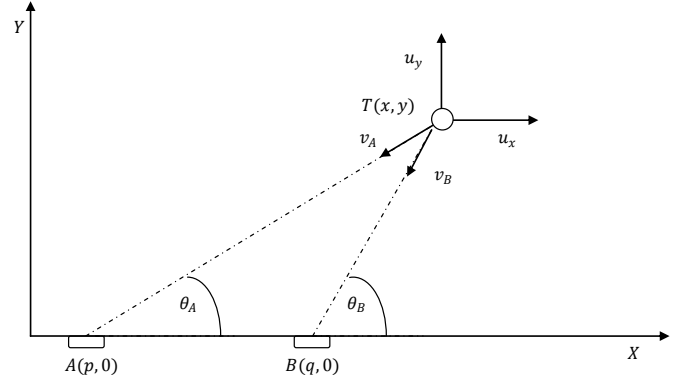


Fig. 2. Sensor Target Locations and Measurements

Using trigonometry it can be derived that, the state of the agent is,

$$\begin{bmatrix} x \\ y \\ u_x \\ u_y \end{bmatrix} = \begin{bmatrix} \frac{q \sin \theta_B \cos \theta_A - p \sin \theta_A \cos \theta_B}{\sin(\theta_B - \theta_A)} \\ \frac{(q-p) \sin \theta_A \sin \theta_B}{\sin(\theta_B - \theta_A)} \\ \frac{v_B \sin \theta_A - v_A \sin \theta_B}{\sin(\theta_B - \theta_A)} \\ \frac{v_A \cos \theta_B - v_B \cos \theta_A}{\sin(\theta_B - \theta_A)} \end{bmatrix}. \quad (4)$$

This holds for only one moving agent. In the case of multiple mobile agents, a data association problem occurs [22].

Doppler discrimination of the mobile agents can be used to obtain the DOA of several mobile agents. Consider the case in Fig.3 where, two mobile agents scatter waves on four antenna elements.  $x_1(t)$  and  $x_2(t)$  are the receiving signals on antenna elements  $Rx_1$  and  $Rx_2$  after the demodulation. They can be expressed as,

$$x_1(t) = k_1 \sin(2\pi f_{d1}t + \phi_1) + k_2 \sin(2\pi f_{d2}t + \phi_2) \quad (5)$$

and,

$$x_2(t) = k_1 \sin(2\pi f_{d1}t + \phi_3) + k_2 \sin(2\pi f_{d2}t + \phi_4). \quad (6)$$

In (5) and (6),  $k_1$  and  $k_2$  are the amplitudes of the incoming waves and here we assumed that they are constant for both waves, due to a very small path difference;  $f_{d1}$  and  $f_{d2}$  are the Doppler frequencies modulated by the two mobile agents;  $\phi_1, \phi_2, \phi_3$  and  $\phi_4$  are the corresponding phases at the receiving antennas.

These two signals are processed individually using Fast Fourier Transform (FFT). The frequency bins are found and the corresponding phases of each frequency bin are obtained. Phase difference of a particular frequency bin  $\Delta\psi_i$ , can be obtained by,

$$\Delta\psi_i = \phi_1 - \phi_3. \quad (7)$$



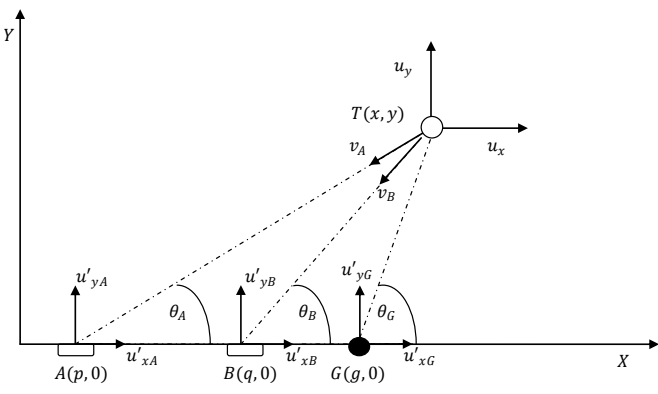


Fig. 5. Tracking a Target when the Sensors are Mounted on a Moving Vehicle

$$\begin{bmatrix} x \\ y \\ u_x \\ u_y \end{bmatrix} = \begin{bmatrix} \frac{q \sin \theta_B \cos \theta_A - p \sin \theta_A \cos \theta_B}{\sin(\theta_B - \theta_A)} \\ \frac{(q-p) \sin \theta_A \sin \theta_B}{\sin(\theta_B - \theta_A)} \\ \frac{v'_B \sin \theta_A - v'_A \sin \theta_B}{\sin(\theta_B - \theta_A)} \\ \frac{v'_A \cos \theta_B - v'_B \cos \theta_A}{\sin(\theta_B - \theta_A)} \end{bmatrix}. \quad (11)$$

where,

$$v'_A = v_A - (u'_{xA} \cos \theta_A + u'_{yA} \sin \theta_A + u'_{xG} \cos \theta_G + u'_{yG} \sin \theta_G)$$

and

$$v'_B = v_B - (u'_{xB} \cos \theta_B + u'_{yB} \sin \theta_B + u'_{xG} \cos \theta_G + u'_{yG} \sin \theta_G).$$

#### IV. MAXIMUM LIKELIHOOD FOR BEARING ONLY LOCATION ESTIMATION

The measurement errors for the sensors are assumed to be mutually independent and Gaussian distributed with zero mean and the same variance.

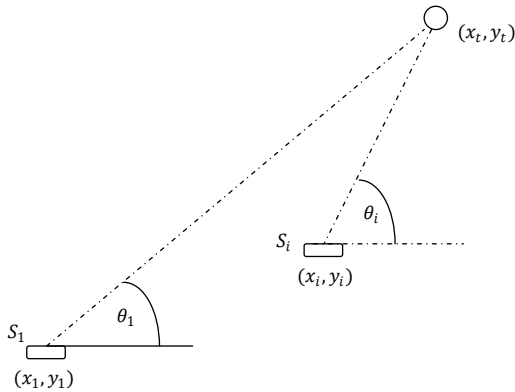


Fig. 6. Sensor Target Locations and Measurements

Bearing only location problem for  $\mathbb{R}^2$  can be formulated as follows. Let  $x = (x_t, y_t)^T$  be the target coordinates vector to be estimated from bearing measurements  $\theta = (\theta_1, \theta_2, \dots, \theta_N)^T$ , where  $(\cdot)^T$  denotes vector or matrix transposition. The target bearings are measured from fixed  $N$  number of sensors at known locations or from sensors fixed to a moving platform where the trajectory of the platform is known. The sensor coordinates are denoted by  $x_s = (x_s, y_s)$ , which associate with the measurement  $\theta_s, (s \in 1, \dots, N)$ . The problem geometry is depicted in Fig.5. The angle measurements consist of the true bearings  $\theta$ , corrupted by additive noise  $\delta\theta = (\delta\theta_1, \delta\theta_2, \dots, \delta\theta_N)^T$ , which is assumed to be zero mean Gaussian with  $N$  covariance matrix  $S = \text{diag}(\sigma_1^2, \sigma_2^2, \dots, \sigma_N^2)$ . Thus the problem is described by the nonlinear equation,

$$\theta = g(x_t) + \delta\theta. \quad (12)$$

where,

$$g(x_t) = (g_1(x_t), \dots, g_N(x_t))^T$$

and

$$g_n(x_t) = \arctan(\Delta_{yn} / \Delta_{xn}),$$

$$\Delta_{xn} = x_t - x_s,$$

$$\Delta_{xn} = y_t - y_s, \quad n = 1, 2, \dots, N.$$

The Cramer-Rao Lower Bound on the covariance of any unbiased estimator for the problem at hand is given by

$$C = (g_x^T S^{-1} g_x)^{-1}. \quad (13)$$

The derivative,  $g_x = \partial g / \partial x$  evaluated at the true target position, is given by

$$g_x = \begin{pmatrix} \frac{-\Delta_{y1}}{r_1^2} & \frac{-\Delta_{y2}}{r_2^2} & \dots & \frac{-\Delta_{yk}}{r_k^2} & \dots & \frac{-\Delta_{yN}}{r_N^2} \\ \frac{\Delta_{x1}}{r_1^2} & \frac{\Delta_{x2}}{r_2^2} & \dots & \frac{\Delta_{xk}}{r_k^2} & \dots & \frac{\Delta_{xN}}{r_N^2} \end{pmatrix}^T$$

where,

$$r_k^2 = \Delta_{xk}^2 + \Delta_{yk}^2; \quad k = 1, \dots, N.$$

$N$  is the number of sensor positions.

The Maximum Likelihood estimator is attractive due to its properties guaranteed by a well known theorem of estimation theory. The theorem states that under mild regularity conditions, the ML estimator is unbiased and its covariance achieves the CRLB, provided that the number of measurements is large enough. If the measurement noise is Gaussian with zero mean, as assumed previously, the ML estimator of the target position  $x$ , is given by

$$\hat{x}_{ML} = \text{argmin} |F_{ML}(x, \theta)|. \quad (14)$$

where, the cost function,  $F_{ML}(X, \theta)$  has the form

$$\begin{aligned}
F_{ML}(x, \theta) &= \frac{1}{2}(g(x) - \theta)^T S^{-1}(g(x) - \theta) \\
&= \frac{1}{2}f^T S^{-1}f = \frac{1}{2} \sum_{n=1}^N \frac{f_n^2}{\sigma_n^2}.
\end{aligned} \tag{15}$$

where,

$$\bar{g} = (\bar{g}_1, \dots, \bar{g}_N)^T = g(x) - \theta.$$

Above equation (14) involves a nonlinear least-square minimization, which can be performed by the Newton-Gauss iterations:

$$\hat{x}_{i+1} = \hat{x}_i + (g_x^T S^{-1} g_x)^{-1} g_x^T S^{-1} (\theta - g(\hat{x}_i)), \quad i = 1, 2, \dots \tag{16}$$

The use of (16) requires an initial estimate,  $\hat{x}_0$ , close enough to the true minimum of the cost function. Such an initial estimate may be available from prior information, or can be obtained from a simple (but suboptimal) procedure. The partial derivatives involved in (16) are evaluated at the current estimated position,  $\hat{x}_i$ . In practice, a number of 2-4 iterations are sufficient for convergence.

## V. SIMULATION

Simulations are carried out to assess the theoretical results. Simulation result in Fig.7 shows the tracking of two mobile agents using three Doppler sensors mounted linearly on the bumper of a vehicle which are 0.5m apart. Fig.8 depicts the same dynamic system with four Doppler sensors mounted linearly. The mean squared error of the two systems are compared in the Fig.9 and it can be seen that the system with the four Doppler sensors performs better than the system with three sensors.

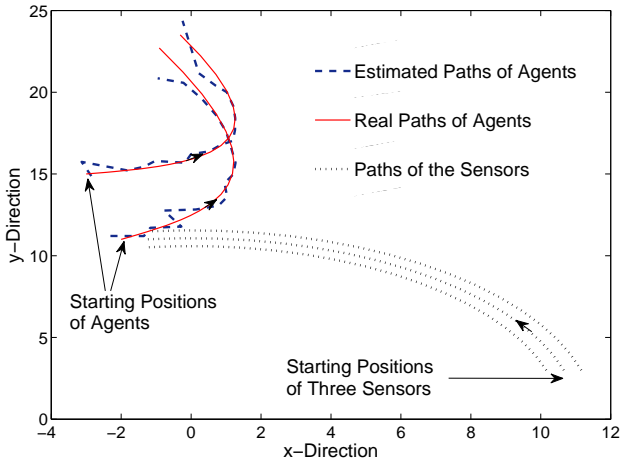


Fig. 7. MLE for Doppler-Radar Tracking with Three Sensors

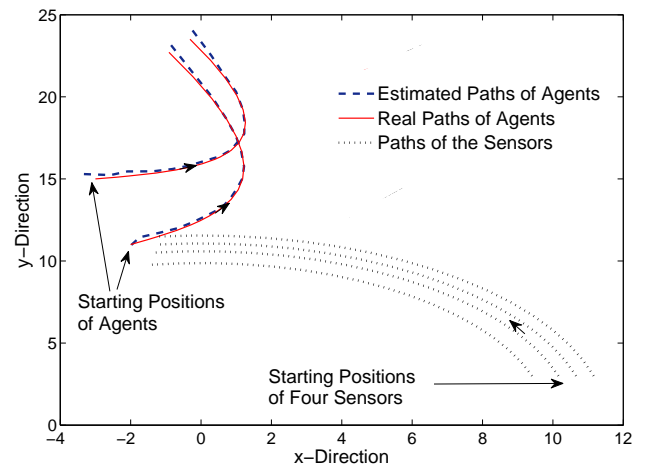


Fig. 8. MLE for Doppler-Radar Tracking with Four sensors

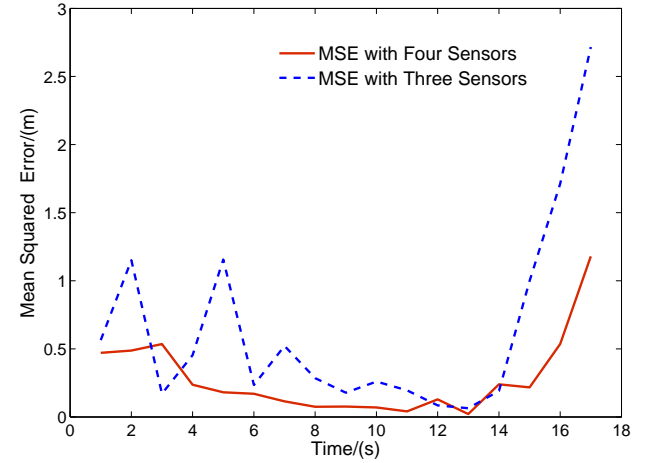


Fig. 9. Variation of Error with Time for Three and Four Sensors

## VI. CONCLUSION

Tracking multiple mobile agents in  $\mathbb{R}^2$  with a Doppler radar system mounted on a moving vehicle has been studied in this paper. Dopplers modulated from the mobile agents on the single frequency continuous wave radar are captured with a linear sensor array and analyzed in order to estimate the positions and velocities of multiple mobile agents. The noisy measurements (assumed to be Gaussian) lead to poor localization, hence, the maximum likelihood estimation (MLE) is utilized to enhance the localization accuracy. The simulation results closely agree with the theoretical derivations.

Maximum likelihood estimates can be heavily biased for small samples and the optimality properties may not apply. Also, maximum likelihood can be sensitive to the choice of starting values. In future studies, model based estimators, such as Extended Kalman Filter (EKF) can be incorporated for better estimation. The uncertainty of all the parameters in this system can be taken in to consideration and study of the effect of the target trajectories with respect to the sensors on the estimation will be a another interesting research

area. Also, data association problem and the optimal linear sensor positioning for better performance are interesting areas for further research.

## VII. ACKNOWLEDGEMENT

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